Renormalization group and its implications for galaxy rotation curves

Davi C. Rodrigues[(a)] , Patricio S. Letelier[(a)], Ilya L. Shapiro[(b)1]

(a) Departamento de Matemática Aplicada, IMECC
Universidade Estadual de Campinas, 13083-859, Campinas, SP, Brazil
(b) Departamento de Física, Universidade Federal de Juiz de Fora, 36036-330, MG, Brazil

Abstract. Quantum effects of matter fields on a classical curved background may lead to the phenomenologically interesting contributions to the action of gravity. The present day knowledge of quantum field theory in curved space leaves the most interesting corrections to the cosmological constant and Einstein-Hilbert terms beyond our possibilities. In this situation one can use phenomenological approach. In the cosmological setting the form of quantum contributions can be established from the covariance arguments. In case of astrophysical applications we meet a bit more complicated situation, and the most challenging problem is to find an appropriate physical identification for the scale parameter of the renormalization group. A very natural choice of this identification provides a remarkably good fit for the rotation curves of a relevant sample of spiral galaxies, without invoking the CDM concept.

The existence of singularities in General Relativity (GR) indicates that our most successful theory of gravity has restricted area of applications. The most natural origin of modifications of GR is the quantum theory, because in the vicinity of singularities one can approach the Planck scale. However, the theoretical realization of the idea of "quantum gravity" is not unique, because the deviations from the gravitational Einstein equations can be either due to the semiclassical corrections, effects of (different models of) quantum gravity, string theory physics, extra dimensions etc.

The corrections to the Newton law is a relatively common feature of the different models of "quantum gravity", including semiclassical approach (see [1] for the introduction and [2] for a recent review). The application of these corrections has been elaborated recently in the cosmological [3] and astrophysical [4, 5] areas.

Let us emphasize that in the most cases the present day state of art in all mentioned approaches to quantum gravity does not enable one to really calculate the relevant quantum contributions to the Newton law in a unique and consistent way. The theoretical estimate for the quantum contribution to the gravitational law involves certain arbitrariness, which can be parametrized by the renormalization group parameter μ from one side,

[1]Also at Tomsk State Pedagogical University, Tomsk, Russia. E-mail address: shapiro@fisica.ufjf.br

and to the physical interpretation of this parameter from the other side. In the present contribution we will summarize the possible effect of the quantum terms, mainly at the astrophysical scale [5].

The standard way of discussing quantum effects in gravity is based on the notion of Effective Action of vacuum (EA). The EA is a generalization of the classical gravitational action at quantum level, which can be seen as a classical action plus quantum contributions. The finite part of EA is a non-local functional, which can not be calculated explicitly, except for the simplest cases [2]. However, in other cases one can use some general features of EA to establish a possible form of low-energy quantum effects at the cosmological and astrophysical scales. Our assumptions include the covariance of EA, and that the low-energy gravity should not have other light degrees of freedom except the ones of the metric.

Under the conditions formulated above, the quantum contributions can be taken in form of a power series in the derivatives of the metric. In the cosmological setting, the second order in derivatives term means there are only the $\mathcal{O}(H^2)$ - like contributions, because the linear in H terms are non-covariant [2, 3]. Then, in the cosmological case, the quantum corrected vacuum energy density $\rho_\Lambda = \Lambda/(8\pi G)$ and the Newton constant satisfy the equations [4]

$$\begin{aligned} \rho_\Lambda &= C_0 + \frac{3\nu}{4\pi}\, M_P^2\, H^2\,, \qquad & \nu &= \frac{\sigma}{12\,\pi}\,\frac{M^2}{M_P^2} \\ (\rho+\rho_\Lambda)\, dG + G\, d\rho_\Lambda &= 0\,, \qquad & \rho + \rho_\Lambda &= \frac{3H^2}{8\,\pi\, G}\,, \end{aligned} \tag{1}$$

where ρ is the energy density of matter, $\mu = H$ and ν is some undefined parameter dependent on the unknown quantum corrections, mainly coming from the particles of the typical mass M. The solution for $G = G(H;\nu)$ can be easily found to be

$$G(H;\nu) = \frac{G_0}{1 + \nu\, \ln\, (H^2/H_0^2)}\,, \tag{2}$$

where $G(H_0) = G_0 \equiv 1/M_P^2$ is the initial value of G.

The formula (2) can be understood beyond the cosmological setting. Indeed, the quantum contributions are assumed to come from the Feynman diagrams which have external legs of a background metric. The relation (2) means we can expect similar relation in the general case, where the Hubble parameter will be replaced by some, yet unknown, combination of the metric components and their derivatives. In this way we arrive at the general relation

$$G(\mu) = \frac{G_0}{1 + \nu\, \ln\, (\mu^2/\mu_0^2)}\,. \tag{3}$$

One can obtain the same formula (3) starting from other arguments. Consider the simplest $\overline{\mathrm{MS}}$-based renormalization group equation for $G(\mu)$

$$\mu\frac{dG^{-1}}{d\mu} \,=\, \sum_{particles} A_{ij}\, m_i\, m_j \,=\, 2\nu\, M_P^2\,, \qquad G^{-1}(\mu_0) = G_0^{-1} = M_P^2\,. \tag{4}$$

Here the coefficients A_{ij} depend on the coupling constants, m_i are masses of all particles of the theory. In particular, at one loop,

$$\sum_{particles} A_{ij}\, m_i\, m_j \,=\, \sum_{fermions} \frac{m_f^2}{3(4\pi)^2} \,-\, \sum_{scalars} \frac{m_s^2}{(4\pi)^2}\Big(\xi_s - \frac{1}{6}\Big)\,.$$

It is an easy exercize to rewrite (4) as $\frac{d(G/G_0)}{d\log\mu} = -2\nu(G/G_0)^2$ and eventually arrive at (3). These consideration shows that, in fact, (3) is not a one-loop, but an *exact* form of the possible relevant renormalization group equation for the Newton constant. Furthermore, we note that the conservation law says that the Appelquist and Carazzone-like decoupling for $\rho_\Lambda(\mu)$ implies a non-decoupling of $G(\mu)$.

An interesting possibility is to apply (3) for description of the rotation curves of the galaxies. Therefore we need the phenomenologically sound choice for μ in the corresponding setting. Different from the previous papers on the subject [6, 4, 7] we consider an identification [5]

$$\frac{\mu}{\mu_0} = \left(\frac{\Phi_{\rm Newt}}{\Phi_0}\right)^\alpha\,, \tag{5}$$

where Φ_0 and α are phenomenological parameters and $\Phi_{\rm Newt}$ is the Newtonian potential computed with the zero boundary condition at infinity. It is important to have α growing fast with the mass of the cosmic object, such that the quantum corrections could become irrelevent within the Solar system.

The result of the application of the Eqs. (3) and (5) can be seen in Fig. 1. As we can see from Fig. 1 and from other plots in [5], the application of the quantum corrections to the galaxies is rather successful. One can see this result as an indication to the possible important impact of quantum (or semiclassical) effects at the cosmic scale.

Acknowledgments

I.Sh. is grateful to CNPq, FAPEMIG, FAPES and ICTP for support. DCR thanks FAPESP for financial support. PSL thanks CNPq and FAPESP for partial financial support.

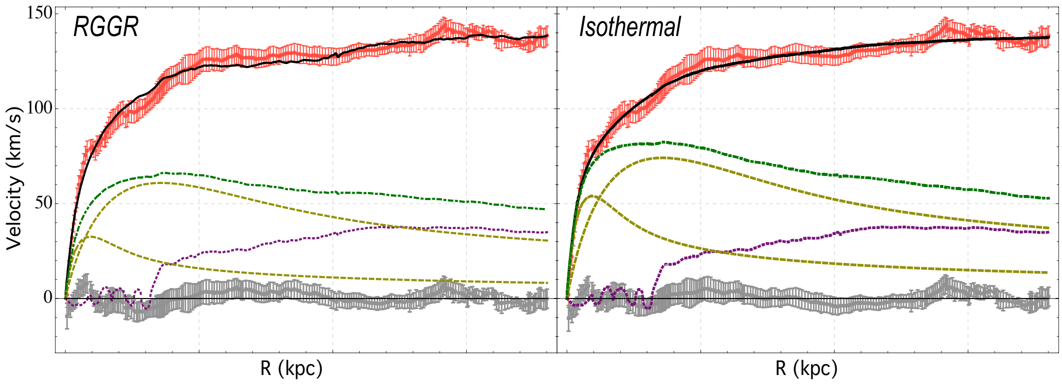

Figure 1: Rotation curve (RC) fits to NGC 2403 (observational data from [8]). The plot on the left shows the resulting RC from our model based on renormalization group corrections, while the one of the right uses one of the standards dark matter profiles. The red dots with error bars are the RC observational data. The solid black line for each model is its best fit RC. The yellow lines stand for RCs decomposition for the stellar part (bulge and disk), the purple line for the gas part and the green for the resulting contribution of the last two parts. See [5] for further details, including other galaxies and comparisons to other models [9].

References

[1] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, "Effective action in quantum gravity," *Bristol, UK: IOP (1992).*

[2] I.L. Shapiro, Class. Quantum Grav. (Topical Review) 25 (2008) 103001.

[3] I. L. Shapiro and J. Sola, JHEP **0202**, 006 (2002); Nucl. Phys. Proc. Suppl. **127**, 71 (2004); Phys. Lett. B **682**, 105 (2009).

[4] I. L. Shapiro, J. Sola and H. Stefancic, JCAP **0501**, 012 (2005).

[5] D.C. Rodrigues, P.S. Letelier and I.L. Shapiro, JCAP 04 (2010) 020.

[6] J. T. Goldman et al., Phys. Lett. B **281**, 219 (1992); O. Bertolami et al., Phys. Lett. B **311**, 27 (1993).

[7] M. Reuter and H. Weyer, Int. J. Mod. Phys. D **15**, 2011 (2006).

[8] W. J. G. de Blok et al, Astronomical J. **136**, 2648 (2008).

[9] M. Milgrom, Astrophys. J. **270**, 371 (1983); J. R. Brownstein and J. W. Moffat, Astrophys. J. **636**, 721 (2006).